Section 14.4

Length of Curves

Supposing the vector-valued function $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ has continuous derivative. We can find a piecewise linear approximation of the curve given by $\vec{r}(t)$.

Example 14.4.1. Find the length of the helical curve

$$\vec{r}(t) = \langle 250\cos t, 250\sin t, 100t\rangle$$

for $0 \leq t \leq 10$.

Example 14.4.2. For a remarkably convenient arc length calculation, consider

$$\vec{r}(t) = \langle 100\cos\frac{t}{100}, 100\sin\frac{t}{100}\rangle.$$

Find the arc length for $0 \leq t \leq t_0$.

To see how Example 14.4.2 can be generalized, consider $\dfrac{ds}{dt}$.

Example 14.4.3. Let $\vec{r}(t) = \langle 2\cos t, 2\sin t, 4t\rangle$, for $t \geq 0$. Find the arc length function $s(t)$. Find a function $\vec{r_1}(t)$ with the same curve as $\vec{r}$ such that the arc length over $0 \leq t \leq t_0$ satisfies $s = t_0$.